Int. J. Heat Mass Transfer. Vol. 17. pp. 381–383. Pergamon Press 1974. Printed in Great Britain

GEOMETRIC BAND ABSORPTANCE OF A NONGRAY GAS WITH ARBITRARY CONFIGURATIONS

S. H. CHAN*

Department of Mechanical Engineering, New York University, Bronx, N.Y. 10453, U.S.A.

(*Received* 18 *March* 1973 *and in revised form* 24 *July* 1973)

A, total area [cm^2];
A_g, geometric band absorptance [cm^{-1}];
A_s, slab band absorptance [cm^{-1}];
A_t, total band absorptance [cm^{-1}];
C_1, integrated band intensity [$cm\ g^{-1}$];
C_2, fine structure parameter [$g^{-\frac{1}{2}}$];
C_3, band width [cm^{-1}];
$e_{b\nu}$, Planck function [$g\ s^{-3}$];
E_1, exponential integral of the first kind;
L_m, mean beam length [cm];
h, characteristic length [cm];
r, distance between two elements [cm];
Pe, effective pressure;
$q_{g\to s}$, radiative flux from a gas body to a spot [$g\ s^{-3}\ cm^{-1}$];
t, pressure broadening parameter;
V, total volume[cm^3].

Greek letters
γ, Euler Mascheroni constant, 0·577216;
κ, absorption coefficient [$cm^2\ g^{-1}$];
μ, cosine of an angle;
ν, wave number [cm^{-1}];
ρ, density [$g\ cm^{-3}$];
τ_h, optical path length ($=\rho C_1 h/C_3$).

CONSIDERABLE complexity of the problems of radiative transfer is attributed to the two kinds of integrations inherent in the equation of radiative transfer, with one being the integration with respect to the radiation frequency and the other the geometry of the problem involved. The most obvious way to avoid the frequency integration is simply to assume the radiating gas as a gray medium. In order to take into account the nongray behaviour, various mean absorption coefficients (such as Planck and Rossenland mean, etc.) were defined which are applicable under certain limited optical conditions [1]. In the past few years, it was found that the frequency integration could be avoided and the nongray effect still could be included in the analysis for all optical conditions by employing the modified emissivity [2] or the total band absorptance of an isothermal gas [3, 4]. Since then numerous correlations have been proposed for the total band absorptance [5–8] and the implementation of the band absorptance in radiative analyses has become a common practice [9–15].

Very recently attempts have been made to eliminate or, at least, to simplify the integration with respect to the geometry. Edwards and Balakrishnan [16–18] have examined the simple case of plane parallel system and concluded that nongray molecular gas radiation could be more compactly formulated in terms of a slab band absorptance rather than the total band absorptance. The slab band absorptance, which is designed specifically for slab geometry, can account for not only the nongray behaviour as does the total band absorptance but also the geometrical characteristics of the slab. Thus the evaluation of the radiant flux, for example, is greatly simplified and lengthy numerical calculations such as those carried out by de Soto [19] can be avoided.

In view of the utility of the slab band absorptance for the problems of slab geometry, it is reasonable to assume the potential utility of other corresponding band absorptances, which will be generally termed as geometric band absorptance, for other geometries, such as circular tube and rectangular shape etc. Therefore, the purpose of the present note is to formulate such a geometric band absorptance for a nongray gas with an arbitrary geometry.

While the total band absorptance is defined by [20]

$$A_t(r) = \int_{\text{band}} (1 - e^{\rho\kappa_\nu r})\, d\nu \tag{1}$$

and the slab band absorptance by [16–18]

$$A_s(r) = \int_0^1 2\mu \int_{\text{band}} (1 - e^{-\rho\kappa_\nu r})\, d\nu\, d\mu, \tag{2}$$

the geometric band absorptance for a gas body with any configuration is defined as

$$A_g(r) = \int_A \frac{\mu_i \mu_j}{\pi r^2} \int_{\text{band}} (1 - e^{-\rho\kappa_\nu r})\, d\nu\, dA_j \tag{3}$$

where r is the distance between two boundary area elements i and j, dA_j is the area of the boundary element j, and μ_i and μ_j are the cosines of the angles between r and the normal to dA_i and to dA_j, respectively. In the case of parallel plates, $\mu_i = \mu_j = \mu$ and $dA_j = 2\pi r^2\, d\mu/\mu$, then the geometric band absorptance reduces to the slab band absorptance.

The striking resemblance of equation (3) to the formulation of the mean beam length suggests the use of the latter for the former. It was pointed out that the slab band absorptance could be employed to find the mean beam length of a slab gas [18]. As a matter of fact, the other way around, namely, the use of the mean beam length to find the geometric band absorptance, is more appropriate here because the mean beam length has been extensively studied and tabulated for many kinds of geometries [21].

Consider an isothermal nongray gas with N non-overlapping infrared bands. Then the radiant flux from the gas

* Now at Polytechnic Institute of New York, Brooklyn, N.Y. 11201.

to a spot is given by

$$q_{g\to s} = \sum_{i=1}^{N} e_{b\nu_i} A_{gi}(r) \quad (4)$$

where the Planck function has been approximated by its value at the band center ν_i since it does not vary greatly with ν within a spectral range comparable to a band width. If the mean beam length of an equivalent hemisphere is indicated by L_m, then the radiant flux can also be written as

$$q_{g\to s} = \sum_{i=1}^{N} e_{b\nu_i} A_{ti}(L_{mi}). \quad (5)$$

Because equation (4) must always be equal to equation (5) for all gases whose band centers may be randomly located, we can conclude that

$$A_{gi}(r) = A_{ti}(L_{mi}) \quad (6)$$

for $i = 1, 2 \ldots N$. It shows that the geometric band absorptance is identical to the total band absorptance with an equivalent mean beam length. Strictly speaking, the mean beam length varies with the path length r and also varies from one band to another. As a general practice, however, a mean value can be used. An extensive list of the mean value of many geometries is available in [21]. In fact the mean value is approximately $L_m - 3{\cdot}5\ V/A$ for all geometries [21]. The use of such a mean value of L_m appears to be reasonable because the mean beam length concepts always yield good results at smaller path lengths while, at larger path lengths, the gas is so opaque that a large error in L_m has little effect on the overall result.

Numerous correlations of the total band absorptance have been proposed [5–8] which can be used in equation (6). For example, the Tien–Lowder correlation [16] which is based on Edwards–Menard piecewise correlations [5] is expressed in a single continuous function:

$$A_t(L_m) = C_3 \ln\left\{\tau_h \frac{L_m}{h} f_2(t)\ \left[\frac{\tau_h L_m/h + 2}{\tau_h L_m/h + 2f_2(t)}\right] + 1\right\} \quad (7)$$

where

$$\tau_h = \rho C_1 h/C_3$$
$$f_2(t) = 2{\cdot}94[1 - \exp(-2{\cdot}6t)]$$
$$t = C_2 P_e/(4C_1C_3)$$

and C_1, C_2 and C_3 have been tabulated for many gases [20]. In the special case of high pressure limit where spectral lines are completely overlapped, a much simpler form [22] can be used,

$$A_t(L_m) = [\ln(\tau_h L_m/h) + E_1(\tau_h L_m/h) + \gamma]\ C_3. \quad (8)$$

For a slab of gas, the geometric band absorptance has been evaluated analytically by using equation (2) and the above limiting form of the total band absorptance. The result is [18]

$$A_s(h) = [\ln(\tau_h) + E_1(\tau_h) + \gamma + 0{\cdot}5 - E_3(\tau_h)]\ C_3. \quad (9)$$

Item A of Table 1 shows the comparison between the dimensionless geometric band absorptance, A_g/C_3, based on equation (9) and that based on the present method which employs a single value of $L_m = 3{\cdot}5\ V/A = 1{\cdot}76\,h$ in equations (8) and (6). Also shown in Table 1 (item B) is the comparison for the case in which the line structure of a band is important. Since no analytical expression is available for this case, exact values of the geometric band absorptance are obtained numerically by using equation (2) and the complete form of the total band absorptance given by equation (7) with $L_m/h = 1/\mu$. It is seen that, in the optical thin limit, the present method underestimates the geometric band absorptance. This is due to the use of the approximate value of the mean beam length ($3{\cdot}5\ V/A$) which is smaller than the exact one ($4\ V/A$). In the optical thick limit, on the other hand, it slightly overestimates because the correct L_m is less than $3{\cdot}5\ V/A$. Nevertheless, the overall agreement is remarkably well. Therefore, unless the exact mean beam length as a function of the optical depth is known, and adoption of a single value of $L_m = 3{\cdot}5\ V/A$ in the geometric band absorptance appears to be practical.

For configurations other than the slab shape, the slab band absorptance is no longer appropriate but the result of the present formulation can be expected to be equally applicable. For example, consider a spherical gas body. Equation (3) becomes

$$A_g = \tfrac{1}{2}\int_0^{\pi} A_t[D\sin(\theta/2)]\sin\theta\, d\theta$$

where D is the diameter of the sphere and A_t is given by equation (7) with D replacing h and $D\sin(\theta/2)$ replacing L_m. Similarly, the exact integration of the above expression is compared with the result of the present method based on $L_m = 3{\cdot}5\ V/A = 0{\cdot}583\,D$ and $L_m = 0{\cdot}63\,D$, the latter being listed in [21]. The agreement of the comparison as shown in Table 2 is as good as the slab case. Since the slab and spherical shapes constitute two extreme geometries, one can conclude the validity of the present formulation in general.

As a concluding remark, it should be pointed out that the geometric band absorptance tends to take the role of the total band absorptance in the radiative transfer analyses [16–18]. The present formulation might provide valuable steps toward the eventual systematical analysis of multi-dimensional problems of radiative transfer.

Table 1. Dimensionless geometric band absorptance of a gas slab

(A) $t \to \infty$, the high pressure limit									
τ_h	0	0·01	0·05	0·1	0·5	1·0	5·0	10	50
Exact [18]	0	0·020	0·095	0·181	0·722	1·187	2·686	3·379	4·989
Present ($L_m = 1{\cdot}76h$)	0	0·018	0·088	0·167	0·677	1·212	2·752	3·457	5·054
(B) $t = 0{\cdot}1$, the band in which line structure is important									
Exact (numerical)	0	0·019	0·088	0·163	0·567	0·879	1·950	2·535	4·044
Present ($L_m = 1{\cdot}76h$)	0	0·017	0·083	0·156	0·569	0·890	1·989	2·584	4·105

Table 2. Dimensionless geometric band absorptance of a spherical gas

(A) $t = 50$									
τ_D	0	0·01	0·05	0·1	0·5	1·0	5·0	10	50
Exact (numerical)	0	0·007	0·033	0·066	0·311	0·578	1·833	2·584	4·383
Present									
$L_m = 0{\cdot}583D$	0	0·006	0·029	0·058	0·276	0·522	1·756	2·523	4·347
$L_m = 0{\cdot}63D$	0	0·006	0·031	0·062	0·297	0·559	1·838	2·612	4·431
(B) $t = 0{\cdot}1$									
Exact (numerical)	0	0·007	0·032	0·063	0·267	0·455	1·232	1·716	3·093
Present									
$L_m = 0{\cdot}583D$	0	0·006	0·029	0·056	0·243	0·422	1·183	1·664	3·047
$L_m = 0{\cdot}63D$	0	0·006	0·031	0·060	0·259	0·447	1·232	1·723	3·119

REFERENCES

1. V. Kourganoff, *Basic Methods in Transfer Problems*. Dover, New York (1963).
2. J. Gille and R. Goody, Convection in a radiating gas, *J. Fluid Mech.* **20**, 47–49 (1964).
3. R. D. Cess, P. Mighdoll and S. N. Tiwari, Infrared radiative heat transfer in nongray gases, *Int. J. Heat Mass Transfer* **10**, 1521–1532 (1967).
4. L. S. Wang, The role of emissivities in radiative transport calculations, *J. Quant. Spectrosc. Radiat. Transfer* **8**, 9233–1240 (1968).
5. D. K. Edwards and W. A. Menard, Comparison of models for correlation of total band absorptance, *Appl. Optics* **3**, 621–625 (1964).
6. C. L. Tien and J. E. Lowder, A correlation for total band absorptance of radiating gases. *Int. J. Heat Mass Transfer* **9**, 698–699 (1966).
7. R. Goody and M. J. S. Belton, Radiation relaxation times for mars, *Planet. Space Sci.* **15**, 247–256 (1967).
8. R. D. Cess and S. N. Tiwari, Infrared radiative energy transfer in gases, *Advances in Heat Transfer*, vol. 8. Academic Press, New York (1972).
9. P. Mighdoll and R. D. Cess, Infrared radiative equilibrium under large path length conditions, *AIAA Jl* **6**, 1778–1779 (1968).
10. J. L. Novotny, Radiation interaction in nongray boundary layer, *Int. J. Heat Mass Transfer* **11**, 1823–1826 (1968).
11. J. L. Novotny and M. D. Kelleher, Conduction in nongray radiating gases, *Int. J. Heat Mass Transfer* **12**, 365–369 (1969).
12. W. P. Schimmel, J. L. Novotny and F. A. Olsofka, Interferometric study of radiation-conduction interaction, *Heat Transfer* 1970, Fourth International Heat Transfer Conference, Vol. III. Elsevier, Amsterdam (1970).
13. J. C. Bratis and J. L. Novotny, Radiation–convection interaction in real gases, Paper No. 72–278, AIAA 7th Thermophysics Conference, San Antonio, Texas, April 10–12 (1972).
14. I. S. Habib and R. Greif, Heat transfer to a flowing nongray radiating gas: An experimental and theoretical study, *Int. J. Heat Mass Transfer* **13**, 1571–1582 (1970).
15. S. H. Chan and C. L. Tien, Infrared radiative heat transfer in nongray non-isothermal gases, *Int. J. Heat Mass Transfer* **14**, 19–26 (1971).
16. D. K. Edwards and A. Balakrishnan, Nongray radiative transfer in a turbulent gas layer, *Int. J. Heat Mass Transfer* **16**, 1003–1015 (1973).
17. D. K. Edward and A. Balakrishnan, Radiative cooling of a turbulent flame front, Paper No. 72-WA/HT-27, ASME Winter Meeting, New York, Nov. 26–30 (1972).
18. D. K. Edwards and A. Balakrishnan, Slab band absorptance for molecular gas radiation, *J. Quant. Spectrosc. Radiat. Transfer* **12**, 1379–1387 (1972).
19. S. deSoto, Coupled radiation, conduction and convection in entrance region flow, *Int. J. Heat Mass Transfer* **11**, 39–53 (1968).
20. C. L. Tien, Thermal radiation properties of gases, *Advances in Heat Transfer*, Vol. 5. Academic Press, New York (1968).
21. H. C. Gottel and A. F. Sarofim, *Radiative Transfer*. McGraw-Hill, New York (1967).
22. S. J. Morizumi, An investigation of infrared radiation by vibration–rotation bands of molecular gases, Ph.D. dissertation, University of California, Los Angeles (1970).